A NAVAL SHIP SCHEDULING ALGORITHM

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THESIS

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ABSTRACT

An algorithm is formulated which schedules ships for at-sea and in-port jobs. The ships are naval warships and their schedules are subject to a variety of constraints. The algorithm allows for individual scheduler preferences and lends itself to future computer implementation.

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I. STATEMENT OF THE PROBLEM

Given a list of ships, a list of jobs, and a list of constraints the problem is to derive a feasible ninety day schedule for each ship.

In the scenario considered here, the ships are naval warships characterized by several factors: identifying information such as name and hull number, grouping information such as division assignment and position within a division such as unit commander, availability dates (Inchop date - the date a ship becomes available for scheduling. Outchop date - the date a ship is no longer available for scheduling), "planned" dates specifying preferred dates for next port call for upkeep and next port call for liberty, and job capability listings - a listing of all jobs that can and cannot be performed by the particular ship. All ships of concern, each with its characteristic information, is one main input into the problem. Job information is the second main input. A job is defined as any activity a ship might be called on to perform. Jobs fall into two main categories - those that require one ship to relieve another ("need-a-relief" or NAR jobs) and those that have no relief requirement ("need-no-relief" or NOR jobs). The first category is exemplified by a patrol type job where a particular station must be occupied at all times. Before one ship can leave that job he must be relieved by another ship. This type job is most prevalent in the problem. The second type job is characterized by port calls for liberty or

upkeep. Here a ship is simply scheduled to go to a particular port. No relief criteria is involved; the only restriction being port availability. Each job also has additional characteristics which include location, the minimum and maximum number of days a ship can perform the job, an indicator which tells whether or not the job is active or inactive on any given day, a group number which identifies the job as a member of a particular group of similar jobs, and finally a number specifying which particular ship is performing the job.

The constraints in the problem apply to both ships and jobs and are of varying degrees of rigidity. Some are basic rules which must always be followed, while others are guidelines which should be adhered to as closely as possible. In the first category are requirements such as the specific minimum and maximum number of days a ship can be on a particular station, the necessity of having a unit commander on a specific job, the ports which will or will not be used as liberty and/or upkeep ports, and particular jobs which must be performed by certain ships. More general guidelines include percentage of time to be spent at sea and in port, preferred steaming speeds to be used in calculating transit times, the maintenance of division integrity where possible, and scheduling as many ships as possible for liberty, upkeep, or outchop through a desired port.

The problem is to formulate a feasible schedule for all ships. An algorithm which accomplishes this formulation is

described here. The complexity of the problem makes it impractical to search for an optimal solution, hence the emphasis is only on satisfying all constraints. Simplified versions of naval ship scheduling problems have been adopted for computer solution, but no attempt has been made to derive an algorithm which will handle the large number of variables resulting from the number of ships, different jobs, and varied constraints in this particular problem. A hand calculation which starts with high priority, restrictive ship assignments and works its way down to filling in the "gaps" with left-over ships is a realistic present-day approach to the problem; an approach which is extremely time consuming and hence restricts consideration of alternative schedules. The approach here is to derive an algorithm which can handle the large number of necessary details of the problem. The long-range plans would then call for a computer implementation of the algorithm, allowing formulation of an optimal schedule which was consistent with the requirements and preferences of the scheduler.

Note, that the problem has been defined in general terms so that it may include any reasonable mix of ships and jobs. The preferences and requirements of the scheduler are a key item which will define and modify the constraints. Part of the problem, therefore, is to maintain a general algorithm which will allow individual judgment to be reflected in the final schedule.

II. FORMULATION OF THE MODEL

A. BACKGROUND

A brief discussion of the typical types of problems scheduling algorithms have attempted to solve will be helpful in understanding the basis for constructing the particular algorithm dealing with the problem stated in Chapter I.

Transportation-type problems are the most common in the field. They represent the typical problem of the minimum cost variety. In a cargo ship scheduling environment the standard transportation problem is "that of a shipper who must find the least cost allocation of moving a commodity from several sources to a number of destinations. The total supply at each source and the total demand at each destination are known, as are the constant per unit cost of transportation between each pair of points."[1] Much work has been done in this area, and the results of various studies are being implemented by shipping companies interested in allocating their cargo fleets in such a manner to either maximize profit or minimize cost while meeting current shipping demands. Variations of the problem investigate optimal size of cargo fleets and specific port schedules for loading and discharging

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[1]Schwartz, N. L., "Discrete Programs for Moving Known Cargoes from Origins to Destinations on Time at Minimum Bargeline Fleet Cost," Transportation Science, v. 2, No. 2, May 1968, p. 134.

ships' cargo, as well as optimal ship schedules. One variation of the problem brings to light another category of similar problems - the assignment problem. Here the problem is "to assign a sequence of cargoes to each ship in the 'best possible' manner. The objective is to maximize the revenue of optional cargoes minus the voyage and fuel costs, under the constraint that all cargoes must be shipped."[2] In its more classical form the problem is to assign resources to jobs in such a way as to maximize the overall suitability of resources to jobs while assigning only one resource to each job.[3] The emphasis in this type of problem is not so much on a scheduling of entities but rather on the assignment of the entities to particular positions.

The "n job, m machine" problem is a second classical type problem in scheduling. The general problem is one in which n jobs are to be scheduled through m machines in such a way that some objective function associated with the process is optimized. One such objective function might lead to a schedule which completes the last job in the least total time; another might concern itself with the minimization of slack-time; a cost criterion would be another prime consideration. Such concepts as random service times could also be incorporated in the model.

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[2] Appelgren, L. N., "A Column Generation Algorithm for a Ship Scheduling Problem," Transportation Science, v. 3, No. 1, February 1969, p. 54.

[3] Dantzig, G. B., Linear Programming and Extensions, Princeton University Press, 1963, p. 136.

A third problem - the airline crew scheduling problem - is another well-known example in scheduling. The basic planning unit is the "rotation" ... a trip flown by a crew which is legal with respect to meeting safety regulations, union requirements and company policy. A rotation is usually a round trip that takes a crew from its home location or "base" and returns it there at the end of the journey. The problem is to select a set of rotations in such a way that each flight segment or "leg" is covered at least once and that the total cost is minimized.[4] This problem of attempting to optimize the allocation of crews to flights has never really been completely solved to the satisfaction of all airlines, but it represents the scope and nature of problems associated with scheduling algorithms.

One reason for describing these various types of scheduling problems is to mention what techniques have become prominent in seeking solutions. The two major methodologies are linear programing and network and flow theory. The suitability of both techniques is almost universal in all types of scheduling problems. Linear programming, in addition to related programming methods such as dynamic and nonlinear programming, are extremely well suited for describing any of the above problems where the objective is to maximize a

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[4] Arabeyre, J. P., "The Airline Crew Scheduling Problem," Transportation Science, v. 3 No. 2, May 1969, p. 141.

function subject to a variety of well defined constraints. The theory of networks and flows is closely related to linear programming and it is often times more convenient to describe the problem in the network framework and apply one of the various well defined algorithms to find an optimal solution.

With such a wide variety of scheduling problems and solution techniques on hand, it might seem reasonable that the particular naval ship scheduling proglem under consideration could be formulated in one, or a combination of, the methods already mentioned. This particular problem, however, does not fit neatly into a general class of problems because it differs in several basic respects. First of all, the major objective is not to find an optimal solution; there is no objective function to be maximized or minimized. Costs, profits, slack-time and/or utility functions are not defined by any type of mathematical function. The concepts of "best" reliefs or "best" assignments is an important one which is made as preferable as possible but no attempt is made to incorporate this elusive concept into any type of objective function; there are too many variable factors involved. The number and variety of constraints are a second basic factor which make this problem unique. As outlined in Chapter I this particular problem incorporates a number of varying types of "rigid" and "guideline" constraints. Formulation of a model which is both suitable for the application of an existing algorithm and which could accurately reflect all the details of this scheduling problem would appear nearly impossible.

The model described here takes a heuristic approach towards finding a feasible solution. . .the schedule is arrived at by working one day at a time, choosing the best allocation of resources for several days into the future based on the situation that day.

B. ALGORITHM DETAILS

1. Arrays for Storing Characteristic and Reference Information

In order to investigate the desirability of assigning a certain ship to a particular job it is necessary to have all the pertinent characteristic information in an easily accessable form. Various types of information will be needed at all stages of the scheduling process and it is mandatory that this information be accurate and up-to-date; in order to expedite the scheduling process the information should be maintained in as meaningful a manner as possible. To incorporate these ideas into this particular problem a variety of arrays are used for information storage. Each one concerns itself with a specific type of data. All references to ships and jobs in these arrays are by number. The actual name of any ship or job of concern, can be obtained from one of two lists, one with all ship names and corresponding numbers, the other with all job names and corresponding numbers. (A layout of all arrays is found in Appendix I.)

The first of the characteristic information arrays is the Ship-Job matrix. Each ship is represented by a particular row in the matrix while each job is represented by a

particular column. All ships and all jobs that are being considered during the ninety day scheduling period are included. The entries in this matrix are boolean - a one or a zero. A one in the (i,j) position represents the fact that the i^{th} ship is capable of performing the j^{th} job; a zero simply means that the i^{th} ship cannot perform the j^{th} job. This information is originally stated among the input characteristics of each ship and is simply a representation of whether a particular ship under consideration is of the right type to meet a job's characteristic specifications. For example, only a destroyer equipped with a certain type of weapon system may be allowed to maintain a particular patrol station; a cruiser might be restricted to reporting to one of a limited number of ports for its upkeep period. The information stored in this matrix is of a permanent nature. To allow for temporary changes or modifications in this data, a temporary Ship-Job matrix is maintained on a daily basis. The initial numerical entries in this temporary matrix are identical to the entries in the permanent Ship-Job matrix. On any given day, however, ones may be turned to zeros and vice versa. These changes reflect special situations. For example, on any particular day a certain ship might be unable to perform a particular job, as is the case when a ship has left the area and is no longer available for assignment. On the other side of the picture, a particular job might be inactive on a certain day. Remembering that all jobs are represented in the Ship-Job matrix, it does not necessarily

follow that all jobs must remain active for the duration of the scheduling period. The temporary Ship-Job matrix reflects the fact that a special job such as an assignment of destroyers to act as carrier escorts might be required for the duration of the third week of scheduling; it would be inactive at all other times.

The matrix which keeps track of such special events is the Calendar matrix. The ninety columns of this matrix correspond to the ninety days of scheduling under consideration. Rows represent jobs and the stored information is either a zero or a one. A one simply means that the job is active, a zero - inactive. This matrix is loaded by the scheduler and on any given day job-columns in the temporary Ship-Job matrix are set to zero if the entry for that job in the Calendar matrix is zero. A one in that same position of the Calendar matrix would have no effect on the corresponding job-column in the temporary Ship-Job matrix; the temporary matrix would continue to hold original or otherwise modified information obtained from the permanent Ship-Job matrix.

The Ship Present Location vector gives an up-to-date display of each ship's station or geographical location identified by number. The name corresponding to the listed number is readily accessible on a separate Location-Number list. In general, at-sea locations are named after their station names while in-port locations are named after their actual port names.

Reference information concerning the steaming time in days between any two locations is displayed in the Distance matrix. Location numbers correspond to the identically numbered row and column numbers in this matrix; therefore, the (i,j) element specifies the number of days (rounded to the nearest whole day) it takes to travel from location i to location j. A standard cruising speed is assumed constant for all transits.

Another characteristic information matrix identifies each ship by division number in the first column and unit commander characteristic in the second column. The second column entry is a zero or a one; a one signifying that the ship, in fact, has a unit commander embarked.

There is one other matrix which deals specifically with ship description information - the Ship Day-Due matrix. Each entry in this matrix is a number corresponding to a date in the scheduling period; the normal range is day one through day ninety, but allowance is made for numbers greater than ninety or less than one in order to take into consideration pertinent characteristic dates that occur after or before the scheduling period of concern. Each date specifies one of the following six planned dates: (1) upkeep date, (2) liberty date, (3) Outchop date, (4) Inchop date, (5) minimum release date, and (6) maximum release date. (A planned date, as defined earlier, is simply a tentative date which is the preferred date for a specific occurrence.) Each column in this matrix corresponds to one of these six planned dates;

each row number corresponds to the identically numbered ship. The upkeep and liberty dates represent the preferred day on which the next upkeep and liberty periods should commence. These planned dates are calculated from input information specifying approximately what percentage of time over a ninety day interval is to be spent at sea, in port for upkeep, and in port for liberty. In-port percentages are interpreted in terms of a specific number of days allocated for that type of job every so many days. An example would be a preference guideline calling for 20% upkeep time which could be interpreted as a six day upkeep period every thirty days. The minimum and/or maximum time constraint for such a job would aid in figuring the actual number of days spent on the particular job; in this example that number is six. (Further details of minimum and maximum job times will be discussed later.) In formulating a ship's schedule one objective will be to have the ship performing its upkeep and liberty jobs on a date as close as possible to the planned date. As soon as a ship commences one of these jobs its next planned date is calculated and entered in the matrix. Outchop and Inchop dates are contained in columns three and four, respectively, for all ships. The use of the labels, Outchop and Inchop, on these dates does not reflect a strict interpretation of the actual naval definitions, but does imply a looser interpretation of non-availability and availability, respectively. The Outchop date for a ship in this matrix is a firmly planned date on which the particular ship

will no longer be available for scheduling. If the schedule concerns itself with ships overseas this might be the date on which the ship would begin preparation for a return to home-port. In most cases a relief would now replace the out-chopping ship. The relief's Inchop date would specify the exact date on which this new ship is available to be scheduled for jobs.

The last two columns in the Ship Day-Due matrix contains minimum release date and maximum release date information. Taken together these two dates give an indication of the time interval within which a particular ship can expect to terminate its present job, the minimum release date specifying the earliest date on which the ship will be considered for reassignment to a new job, the maximum release date specifying the most distant date that a particular ship would be expected to remain at its present job. These dates are calculated for a ship as soon as it commences a particular job. Each job has associated with it a minimum and maximum number of days a ship is expected to remain on that job. This information goes into the calculation of the dates entered in the last two columns of the Ship Day-Due matrix. The mean of the minimum release date and maximum release date is used to define the target release date. The algorithm attempts to terminate jobs for ships as close as possible to the target release date. For ships that are performing jobs that do not need a relief, such as upkeep and liberty jobs, a slight

variation of the above definitions applies. The minimum release date for such jobs represents the fact that on that date the ship is available for assignment; the job time has exhausted and since no relief is necessary the ship may now be scheduled for a new job. In this context, maximum release date becomes meaningless so it is arbitrarily given the same date as the minimum release date. The algorithm does not try to prematurely schedule a ship for a new assignment before its liberty or upkeep time has expired; the ship simply becomes available on its minimum release date and is then ready for assignment.

The final two matrices which hold characteristic information concern themselves with jobs. The first, the Job Report matrix, holds seven pieces of job information in each of its seven columns; the i^{th} row corresponds to the i^{th} numbered job. The first column contains the job location numbers; the second column contains boolean information specifying whether the job is active on the day of concern (1) or inactive (0); the third column contains boolean information specifying whether the job is one that does not need a relief (1) or one that does need a relief (0); the fourth column gives the number of the ship presently performing the job (a zero indicates no ship); and the fifth column gives the "group number" of the particular job. The "group number" identifies all jobs that can be considered part of one overall or common job. Since a job has been defined as being able to be performed by one and only one ship, it is necessary to be

able to collectively refer to a group of jobs, such as three jobs that together make up a certain patrol station. All three jobs in this one patrol station would have the same group number. Likewise, all upkeep jobs in the same port would have the same group number. The total number of ships in the n^{th} group can be found by observing the n^{th} element in a separate Group Count vector. The fifth and sixth columns of the Job Report matrix specify the minimum and maximum number of days, respectively, a ship would be expected to perform that job. It is this information which is used as input into calculating a ship's minimum and maximum release dates in the Ship Day-Due matrix.

2. <u>Making Assignments</u>

a. The Relieve Factor for NAR Jobs

The Relieve Factor is a measure of the desirability of assigning a certain ship to a certain job. It consists of two components - the Acquire Factor and the Release Factor. The Acquire Factor focuses its attention on the ship that is available to be assigned; the Release Factor concerns itself with a ship that is on a job. The desirability of having a certain ship relieve a certain other ship on a particular job is assumed to be a linear combination of the Acquire Factor and the Release Factor. Their sum is the Relieve Factor.

If a ship is available for assignment, it is assumed that it would be preferable to assign it to the closest of all feasible jobs. Hence the number of days

travel to a particular job adjusted by a weighting factor is one element of the Acquire Factor. Since one objective of the scheduling algorithm is to assign ships to upkeep, liberty and outchop on days as close as possible to the planned dates, i.e., the dates referenced in the Ship Day-Due matrix, it becomes less and less desirable to assign an available ship to a NAR job as one of these planned dates draws near. Therefore an additional three elements of the Acquire Factor reflect the "undesirableness" of assigning a particular ship to a NAR job as one or more of these three planned dates does in fact approach. The fifth element of the Acquire Factor expresses the desirability of assigning an available ship to a job within a job group which has members of the available ship's division performing jobs in that group. The idea here is the preference for establishing division integrity; it is desirable to have as many ships as possible of the same division operating together.

Note that the emphasis has been on desirability and undesirability. The Acquire Factor involves five such elements. Each element is assigned a numerical value and their sum is the value of the Acquire Factor. The value of the first element is simply minus the number of days travel adjusted by the weighting factor. The second, third, and fourth element values are obtained from linear functions. When the upkeep, liberty, or outchop planned date is a specific number of days away, the value of the element is minus an integer value. The value is decremented each day and

arbitrarily truncated at a terminal negative value after the available ship is a given number of days overdue for upkeep, liberty or outchop. Thus as the present date gets closer and closer to one of the planned dates, the corresponding element's value gets more and more negative and continues to grow for every day overdue until some arbitrary limit. If any one of the planned dates is such that the present date minus the planned date yields a number greater than the corresponding function's upper bound, then that element's value is zero. Thus the algorithm does not start taking into consideration planned upkeep, liberty or outchop dates until some set number of days in advance of those dates. The value of the fifth element in the Acquire Factor is calculated by adding a constant value adjusted by a weighting factor if by being assigned to a particular job, the available ship will be establishing division integrity. For each ship of its own division which it will be joining, the fifth element's constant value is increased. If the available ship does not establish division integrity, the fifth element's value will be zero.

The Release Factor is composed of four similar elements. The value of each element is specified by a constant or weighting factor times the number of days a ship on a job is overdue for relief, upkeep, liberty, and outchop respectively.

Since the Relieve Factor is a measure of desirability of assigning a particular ship to a particular job,

it is clear that overall desirability will be a combination of the available ships "availability" - i.e., closeness to the job, lack of planned commitments in the near future, and preferences for joining other members of its division - and the "need-to-be-relieved" for the ship on the particular job under consideration resulting from that ship being overdue for planned commitments. Hence the Relieve Factor is the sum of the Acquire Factor and Release Factor. On a given day Relieve Factors are calculated for all combinations of available ships and their respective feasible jobs. Relief assignments are made starting with the most positive Relieve Factor since this indicates the most desirable assignment. Assignments are continued to be made until there are no more positive Relieve Factors.

b. <u>Storage of Acquire and Release Factor Values</u>

The list of values given to the various elements in the Acquire and Release Factors are located in arrays. The second, third and fourth elements of the Acquire Factor each has a particular matrix where the first column has a listing of numbers which represent days within the present date for which upkeep, liberty and outchop dates have been planned. The second column gives the associated value of the element. An example is given in Figure I. The weighting factor for the first Acquire Factor element, number of days travel, and the weighting factor for the fifth element, desirability of establishing division integrity, are recorded in a

	Number of Days Within Which Upkeep is Planned	Element Value
	5	-1
	4	-2
	3	-3
	2	-4
	1	-5
OVERDUE	0	-6
↓	-1	-7
	-2	-8
	-3	-9

Figure I

TYPICAL ARRAY FOR ACQUIRE FACTOR
SECOND ELEMENT VALUES

two element vector. The values for each of the four elements in the Release Factor are also stored in vector form. Having all these values in separate arrays will allow for ease in adjusting values to reflect the scheduler's preferences.

c. The Relieve Factor for NOR Jobs

Thus far the discussion of the Relieve Factor has been under the assumption that there is both an available ship and a ship on the particular job considered for assignment. Consideration must also be given for assigning an available ship to a job that does not need a relief such as the upkeep,

liberty, and outchop jobs. In light of the previous discussion it is clear that for a particular ship, as the planned date for one of these NOR jobs approaches or has past, the Acquire Factor for that ship becomes more and more negative, ensuring that it is not sent to a NAR job, i.e., the Acquire Factor will be more negative than the Release Factor is positive causing their sum, the Relieve Factor, to be less than or equal to zero. At this point a new Relieve Factor is calculated for each of the remaining available ships which were not assigned to a "need-a-relief" job. This second calculation will again express desirability of sending a ship to a job but only to "need-no-relief" jobs.

Initially, all Relieve Factors are set to an arbitrarily high positive constant, +100. Assignments will be made by picking the smallest Relieve Factor calculated in the following manner: First upkeep jobs are investigated. For each upkeep job the steaming time to the location of that job is added to the present date. This date is compared to the planned upkeep date. If it is greater than or equal to the planned date, the Relieve Factor is given a value equal to the planned date minus the present date plus the number of days steaming time. What this method is doing is noting if the planned upkeep date is on a date which is less than or equal to the present date plus the number of days it will take to get to the particular upkeep port. If the upkeep date falls within this range a Relieve Factor is calculated,

otherwise it is left at its initial value of 100. A simple example might help clarify this concept. If the present date is day 5 and the travel time to the particular upkeep job under consideration is 3 days, then the algorithm considers sending to that job any ship whose planned upkeep date is less than or equal to 8. If the planned date is eight, the ship will arrive exactly on this date. If the planned date is less than eight the ship has become overdue for upkeep or at least will be overdue by the time it arrives at the upkeep job. Since the available ship that is most overdue for upkeep should be assigned first, this ship will be the one with the smallest Relieve Factor. If in the above example one ship's planned upkeep date is day 1, its Relieve Factor would be: 1 (planned upkeep date) - 5 (present date) + 3 (travel time) = -1; a second ship with a planned upkeep date of 6 would have a Relieve Factor equal to eight (6-5+3=8). Hence if these were the only two Relieve Factors under consideration, the first ship would be assigned first and, if possible, the second ship would then be assigned. Note that a ship with planned upkeep date of 9 would not be considered for assignment to this job because the present date (5) plus travel time (3) is less than its planned date (9). Its Relieve Factor would maintain its initialized value of +100 indicating "no-consideration."

After considering each upkeep job in this manner, the same procedure is used for calculating Relieve Factors for

liberty jobs and outchop jobs. After all possible Relieve Factors have been calculated the process of making assignments begins. The smallest Relieve Factor is picked out and the corresponding ship is assigned to the corresponding job. The process continues until only +100 Relieve Factors remain.

d. The Relieve Factor Matrix

In order to keep track of all Relieve Factors that are calculated a special matrix is used. This matrix has a row corresponding to each ship in the problem and a column corresponding to each job. Two additional columns contain information that is characteristic of each ship for any given day; one column has boolean information indicating whether the ship is available for the day of concern (1) or not available (0); the second column holds a number for each ship indicating how many total ships in that ship's division are available for that day. As has been implied in the above discussion the data in the Relieve Factor matrix is of a temporary nature; it is recalculated daily. Before calculating individual Relieve Factors for ships being considered for "need-a-relief" jobs, all entries, except those in the two special columns previously mentioned, are set equal to a large negative constant -100. As Relieve Factors are calculated they are entered in the appropriate row and column location. When it comes time to make assignments with this first set of calculated Relief Factors, the matrix is scanned for the most positive entry. After the assignment has been made all entries in the row and column of the assignment are

set equal to zero, indicating that the ship is no longer considered for other assignments, and the job is no longer considered in need of a relief. If after all possible assignments have been made, available ships are still present, then all elements are reset to a large positive constant, +100, and the second set of Relieve Factors are calculated to investigate possibilities of sending these ships to "need-no-relief" jobs. Again, after all new Relieve Factors have been entered, the assignment processes commences and after each assignment the corresponding row and column entries are all set to +100 indicating no further considerations will be made for that ship or job.

e. The History Matrix

One final matrix is of utmost importance in the formulation of the scheduling algorithm for it keeps track of the daily activities of each ship and does in fact represent the final product of the algorithm - the schedule. Each ship's activities are recorded in the appropriate row of the History matrix; column numbers correspond to day numbers. Initially, all entries are set to zero. Initial ship input characteristics are interpreted and appropriate starting jobs or activities are placed in the History matrix. If a ship is performing a job, the job's number is entered in column one for the ship. Similarly, if a ship is in transit a "T" appears; if a ship is available but is performing no job an "A" appears; if the ship is actually not present (not under consideration for scheduling) a zero appears. On a daily

basis the elements in the matrix are updated. Reliefs and assignments are reflected by the appearance of new job numbers and/or "T"'s representing "in transit." If a ship becomes available, but is not required for immediate assignment, an "A" appears in the matrix. Hence for each day, each ship's activity is entered. After ninety days of scheduling the algorithm ends and the History matrix represents the formulated schedule.

III. ALGORITHM PROCEDURE

To see how the arrays described in Chapter II are used, a step by step explanation of the scheduling algorithm follows. The logic flow chart in Appendix B will be helpful in following the procedure described.

The data which is specified by the scheduler and used as input into the algorithm is: (1) each ship and its corresponding eleven characteristics listed in Chapter I; (2) each job and its corresponding eight characteristics also listed in Chapter I (NAR jobs are first, followed by NOR jobs); (3) the number of the last NAR job, the number of the last liberty job, the number of the last upkeep job (liberty, upkeep, and outchop jobs are each grouped within the NOR jobs); (4) the number of days transit time between each location and all other locations; (5) the number of jobs in each group of jobs; (6) the arrays of element values for each of the five elements in the Acquire Factor and each of

the four elements in the Release Factor; (7) the planned number of days between upkeep jobs and liberty jobs respectively; and (8) the Calender indicating active and inactive jobs for each day in the scheduling period.

The first step in the algorithm is to initialize all elements of the History matrix and Ship-Job matrix to zero. After this step all the input data is read and stored in its appropriate matrix. Arrays which will be filled by this procedure are as follows: the Ship-Job matrix, the Present Ship Location matrix, the Distance matrix, the Division matrix, the Ship Day-Due matrix, the Job Report matrix, the Job Group Data matrix, the Calender matrix, the Acquire Factor element matrices dealing with upkeep, liberty, and outchop, the Acquire Factor vector for elements one and five of the Acquire Factor, the Release Factor element value vector, and the vector specifying the number of days between upkeep jobs and the number of days between liberty jobs. Using this initial data, as much as possible of the History matrix is filled; every ship is accounted for on day-one and its activity is entered in this matrix. A ship is either on a job, is not available, or is available; no assignments made prior to day-one are considered in effect, thus no ship is in transit on day one; it is simply characterized as available for assignment. The date is now officially set to day one and the actual scheduling procedure begins.

The temporary copy of the Ship-Job matrix is made. The data is gained from the original Ship-Job matrix. Columns -2 and -1 of the Relieve Factor matrix are set to zero; all other elements in this matrix are set to -100.

Now available ships are located and a "1" is entered in the -2 column of the Relieve Factor matrix for each such ship. Available ships are found by examining each NOR job, making sure it is active, noting if a ship is presently performing the job, checking the minimum release date for such a ship, and marking it available if its minimum release date is less than or equal to the present date. The present date column of the History matrix is also scanned for "A"'s and corresponding ships are flagged with a "1" in the -2 column of the Relieve Factor matrix.

Next, the temporary Ship-Job matrix is revised to reflect the present day situation. Jobs which are designated inactive in the Job Report matrix cause complete corresponding columns in the Ship-Job matrix to be set to zero. For each remaining job that a ship can feasibly perform, as designated by a "1" in the matrix, the arrival date is calculated and compared to the minimum release date of the ship presently on the job. If the arrival date is less than the minimum release date, a zero is entered in the temporary Ship-Job matrix for that combination. If the arrival date is greater than the minimum release date or if no ship is on the job under consideration then the "1" is maintained in the matrix indicating the

particular job is a feasible one for the available ship. The implication here is that no available ship will be allowed to relieve a ship on station before the later's minimum release date.

Relieve Factors are now calculated for each combination of available ship and each of its feasible jobs. NAR jobs are considered first. The Acquire Factor is calculated. The number of days travel is ascertained from the Distance matrix and multiplied by its weighting factor found in the appropriate information vector. Upkeep, liberty, and outchop element values are picked out of the appropriate matrices if the planned date for such activity lies within a certain range of days described for each element. The group number of the particular job under consideration is located in the Job Report matrix. For other ships on jobs with the same group number, the division number of each is compared to the division number of the particular ship under consideration. For each identical division number the Acquire Factor's fifth element value is incremented by the designated constant.

The Release Factor is now calculated for the ship-job combination under consideration. If the ship presently on the job is overdue for relief, upkeep, liberty or outchop as designated in the Ship Day-Due matrix, the number of days overdue is multiplied by the proper weighting constant listed in the Release Factor element value vector. The Relieve Factor is now set equal to the sum of the Acquire Factor and

Release Factor values and entered in the appropriate location in the Relieve Factor matrix. This procedure repeats itself for all available ships. If there are no available ships on a given day the schedule for all ships remains fixed and the date is incremented to the next day.

Acquire Factor and Release Factor element values are a key concept in the algorithm. For elements whose value varies over a particular range of days, the final schedule can be made to reflect the individual preferences of the scheduler by changing the range of days for which a particular element has a value, or by changing the values associated with each particular day in the range of days. For elements whose values are constants the same result may be obtained by changing the weights associated with the particular values. Looking first at the Acquire Factor, for example, it is evident that the degree of availability of a ship can be reduced by assigning a very large negative value for any particular element corresponding to a planned date occurring within a certain range of days. If the scheduler wants to put a high priority on available ships arriving at their upkeep ports on their planned upkeep dates, then all values in the Acquire Factor upkeep element matrix would be given large negative values or the range of days in the matrix could be expanded. Large negative values assigned four, five, or six or more days in advance of the planned upkeep date would ensure a ship not being assigned to a NAR job and, hence, remaining available for the upkeep job. Likewise, if division integrity

is judged an important criteria for assigning ships, then the schedule can be made to more heavily reflect this criteria by giving the fifth element of the Acquire Factor a heavier weight. The relative values assigned to the five elements in the Acquire Factor are therefore important in indicating which element or elements are considered most significant. In addition, the values must also be judged relative to the values of the Release Factor elements because it is the sum of Acquire Factor element values and Release Factor element values which gives the overall priority of assignment. These same characteristics, therefore, must necessarily hold true for the Release Factor element values. Given that there available ships, a relief can be forced to take place on a date very close to a target date by assigning a high positive value for the particular Release Factor element of concern. This element value must also be high relative to element values in the Acquire Factor, thus ensuring a highly positive Relieve Factor and putting a high priority on relief of the particular ship of concern. Thus it is seen that the assignment of weights and element values in the algorithm is a powerful method for allowing the generation of schedules which incorporate individual preferences.

The next major part of the algorithm deals with an investigation of division integrity. If ships of the same division are available, it is preferable to maintain division integrity by sending as much of the division as possible to a single job group. (Remember that a job group consists of individual jobs that when considered together represent one overall job,

five jobs which together make up a certain patrol, for example.) The way the algorithm investigates and compensates for maintaining division integrity is as follows: The first ship flagged as available in the -2 column of the Relieve Factor matrix is noted. Any other ships with the same division number as this first ship are located and that group of available ships is given a group number. The group number is entered in column -2 of the Relieve Factor matrix for each appropriate ship and the total number of ships in the group is entered in column -1. All other available ships are similarly investigated. A group number will be assigned to every ship, even if a ship is the only member of a group.

Now that both the total number of available ships of the same division and the particular ships within the division have been indicated in the appropriate columns in the Relieve Factor matrix, the feasibility of assigning a ship group with two or more available ships to a job group with two or more individual jobs is investigated. The job group must have an equal number or greater number of individual jobs than there are available ships in the ship group. When such a job group is found, the algorithm references the temporary Ship-Job matrix and tries to make a feasible assignment of ship group to job group. If such an assignment can be made, a constant bonus factor is added to the value of the Relieve Factor already calculated for each particular ship-job combinations. This bonus factor will be reflected only on those Relieve

Factors which correspond to the exact assignments which make up the calculated feasible combination. Each job group which is greater than or equal to the ship group is investigated. If a feasible assignment cannot be made, then no bonus is added. After all job groups have been considered assignments of the next ship group with two or more available ships of the same division are investigated. Note that no actual assignments are made in this process. Only ship group-job group feasibility is investigated; the bonus factor for maintaining division integrity is added if and only if a ship group can be successfully matched up with a job group. There is no limitation as to the number of job groups to which a ship group can be matched.

When this process is completed all available ship groups have been investigated, the Relieve Factor matrix entries now indicate desirability of every ship for every job. Even ships that were not flagged as available will have entries in their respective rows; the entries will be the initial values given to all elements in the matrix, -100.

The algorithm next specifies that assignments will be made for the NAR jobs only where Relief Factors are greater than zero. The most positive (i,j) element in the matrix is chosen first, and ship i is assigned to job j. The transit time and day of relief are immediately calculated by referencing the Distance matrix, and the proper entries are made in the History matrix to reflect the assignment. All i^{th} row elements and all j^{th} column elements in the Relieve

Factor matrix are set to zero indicating the i^{th} ship and j^{th} job are no longer under consideration for assignment and relief respectively. Now the remaining most positive (i,j) element is considered and the identical procedure takes place. This process continues until there remains no Relief Factors greater than zero; all assignments have been made for NAR jobs.

"Need-no-relief" jobs are considered next. All elements in the Relieve Factor matrix except those in column -2 and -1 are initialized to +100. Remaining available ships still have positive numbers in the -2 column; this element for ships already assigned was turned to zero immediately following their assignments in the procedure just explained. Relieve Factors are now calculated in the manner explained in Chapter II. This calculation assigns the largest negative Relieve Factor value to the ship longest overdue for a particular NOR job. If no ship is overdue for such a job, then consideration is given for sending a ship to a job which has a planned date less than or equal to the present date plus the number of days transit to that job. Remaining ships which fall into neither of the above categories are simply not given any assignment and remain available for another day. These various conditions are reflected in the value of the elements found in the Relieve Factor matrix for the available ships. Assignments are made starting with the most negative or smallest element value; the particular ship is assigned

to the corresponding job and the transit days and commencement day for the job are recorded in the History matrix. Assignments are continued to be made while there exists Relieve Factors less than or equal to some positive constant; the positive Relieve Factor representing the case where a ship is being assigned to a NOR job because of the closeness of a planned date for such a job.

It is evident from the method described above that the algorithm does not concern itself with making the most number of assignments possible. If the number of assignments was the criteria for making assignments then all possible combinations of available ships and their respective feasible jobs would be investigated. Ships assigned first would be those that were most limited in the number of jobs they could feasibly be assigned to on a given day. By working down the list of available ships in such a manner that the most flexible ship was considered last, and then investigating various alternative solutions, the maximum number of assignments would be made. However, this method does not give top priority to making assignments to "most needed" jobs, where "most needed" implies jobs where the ships are longest overdue for relief or in-port assignment. This priority of assignments is a more important consideration than just the number of assignments, and is therefore the basis for making assignments in the algorithm. Eventually, all available ships will be assigned to new jobs; the algorithm's assignment criteria ensures assignments to the "most needed" jobs.

All assignments having been made, the algorithm next calls for the date to be incremented by one day and all arrays brought up-to-date. Ships that have not been relieved on station are given their same job numbers in the History matrix. In the same matrix, ships that have now completed upkeep and liberty jobs are marked as available and new upkeep or liberty planned dates are calculated and entered into the Ship Day-Due matrix. New minimum and maximum release dates are calculated for ships that have made reliefs, while those ships coming off their jobs are marked available for new assignment in the History matrix. All new ship locations are indicated in the appropriate Location matrix, and the Job Report matrix is updated to reflect any new ships on a particular job. Ships that have outchopped are indicated not available by zeros in the History matrix and ships that have inchopped are marked as available; this latter information is gained by scanning the appropriate column in the Ship Day-Due matrix.

At this point the row corresponding to the date in the Calendar matrix is entered into the second column of the Job Report matrix. This action gives the latest status of each job with respect to being active or inactive. Now a new temporary daily copy of the original Ship-Job matrix is made and then immediately modified to reflect the latest job status report. The Relieve Factor matrix is again initialized, available ships are flagged and the whole scheduling algorithm is ready for another iteration.

IV. EXTENSIONS OF THE ALGORITHM

The algorithm has been formulated to incorporate the basic elements of a naval ship scheduling problem. There are several areas in the scheduling problem which could be given more detailed attention.

First of all, travel times could be computed for several different transit speeds, therefore changing the number of feasible jobs a ship could perform. Most assignments could still be evaluated in terms of travel time using normal cruising speeds but, by allowing for the option of faster transit times, overdue situations could be corrected in quicker fashion. Reliefs could be made closer to a ship's target release date, and in-port jobs could commence on dates closer to planned dates.

Ship preferability for a particular job should also be a consideration for making assignments. Rather than simply classifying a ship as capable or not capable of performing a certain job, it would be advantageous to have a range of capability. A ship might in fact have the characteristic necessary for maintaining a certain station, but perhaps a ship with a newer, more advanced radar could do a better job on the particular station. If both ships are available, preference for assignment to the job should be given to the latter ship. This concept could be included as another element in the Acquire Factor where a high positive value might reflect strong preferability for a particular ship-job combination;

lower values would simply imply less preferability.

Liberty ports and liberty time also deserve closer attention. Oftentimes a ship's schedule would want to ensure at least one visit to a particular liberty port; this is particularly true in terms of overseas liberty ports. Also there is usually a preference for liberty time to be scheduled over weekend or holiday periods. Limiting entry to liberty ports on particular days can be carried out in the algorithm as it presently stands,but additional sophistication would have to be incorporated to assign ships to optimal temporary jobs while waiting for port entry.

The algorithm incorporates the concept of division integrity and investigates the possibility of keeping together a group of available ships of the same division, but the result of the investigation is an "all or nothing" result; that is, if the group can be kept together a bonus is assigned, if not, no bonus is assigned. Ideally, consideration should be given to keeping as much of the group as possible together. This would necessitate investigation of all combinations of ship groups and partial ship groups with each equal size of larger size job group. For a schedule dealing with a large number of ships and jobs this investigation would become extremely time consuming and of questionable worth.

An additional constraint which might be included would limit the number of times a ship can perform a certain job. This constraint would be particularly applicable where the

scheduler wants to avoid reassigning a ship to a particular at-sea station or a second port call at the same liberty port. Variety is an important criteria in formulating schedules which are as pleasing as possible to the individual ships concerned.

Sophistication of the Acquire Factor and Release Factor functions might also contribute to a better schedule. The element values might more closely approximate a scheduler's preferences if the size and weights of values were derived from functions that were not linear functions. Selection of the "best" functions would necessarily involve a great deal of research and trial-and-error procedures. Probability distributions could also be associated with target dates and thus the probability of relief for a given interval of days surrounding the target date could be estimated.

Some type of cost criterion is certainly a consideration which the algorithm could be expanded to include. If operating cost were a prime consideration, then steaming times would be kept to a minimum. This aspect of cost can be handled in the model's Acquire Factor by assigning a high weight to the penalty paid for transit times. More time between upkeep periods and less time between liberty or other in-port periods might be an additional solution. Actual operational costs in performing certain jobs would be required as inputs into the model in order to fully investigate schedules that would attempt to minimize the total cost of ship operations; minimum operation levels would also have to be specified.

A slightly more complicated method of making assignments, but one which incorporates all the ideas of the present algorithm, would include a concept called "backtracking." On any given day all available ships would be simultaneously assigned to all feasible jobs. The algorithm would then proceed to the next iteration; jobs would be assigned to the first ship to arrive at the job location. Remaining ships would stay in transit to remaining unrelieved jobs. The process would continue until a day on which there was no available ship to fill a particular job. The model would then backtrack to the last assignments made, make alternative assignments, and then proceed forward again. Should a blocking point occur at the same location as before, then the model would backtrack further than the first time, make alternative assignments, and then proceed forward again. This process would result in a schedule which is more optimal than the method described here.

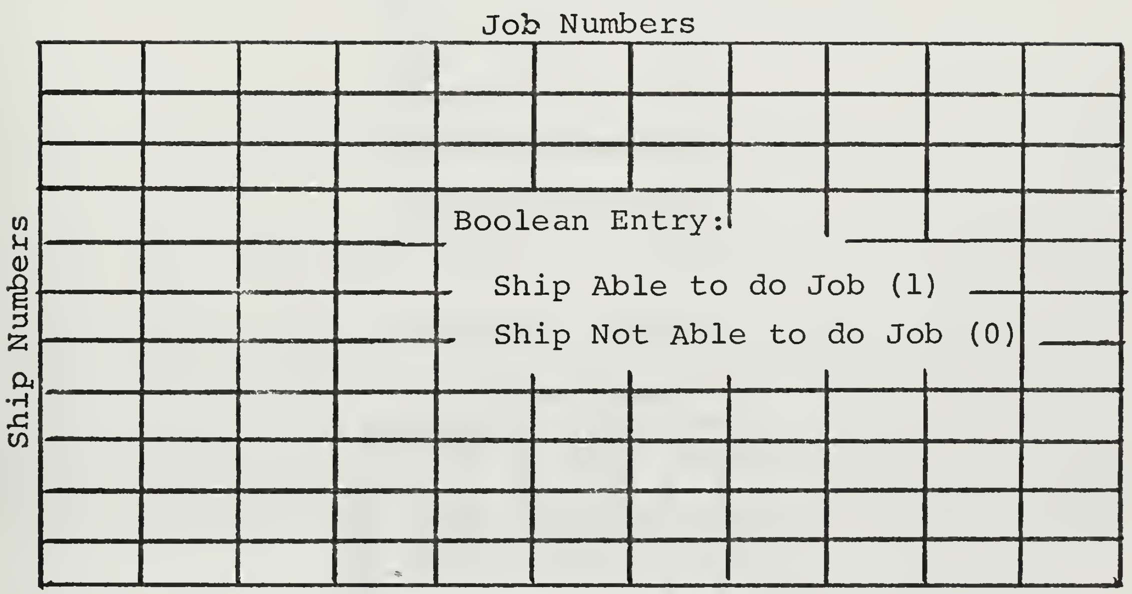
The entire algorithm as it presently stands has been formulated with the idea of computer implementation in mind. It is a complicated problem because it involves a variety of constraints. Obtaining a solution becomes very time consuming as the number of ships and jobs increases. The algorithm, however, does break the problem down so as to take into consideration as many constraints as possible and to lend itself to computer solution. Once programmed, the scheduler should have a powerful tool for formulating schedules which satisfy the numerous constraints and reflect the individual's preferences.

APPENDIX A

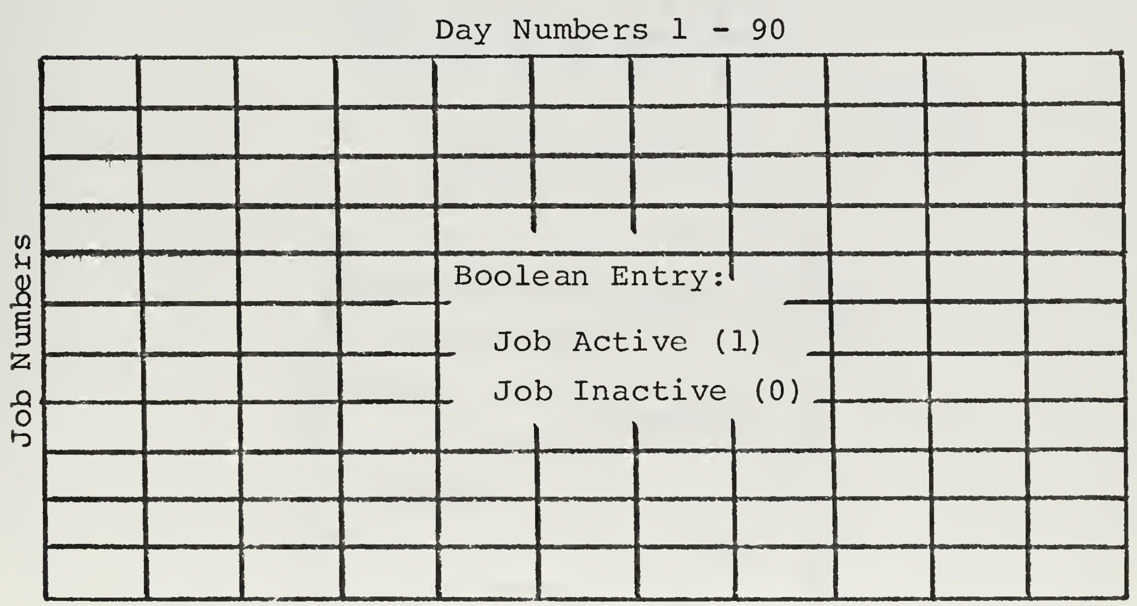
DATA ARRAYS

SHIP-JOB MATRIX

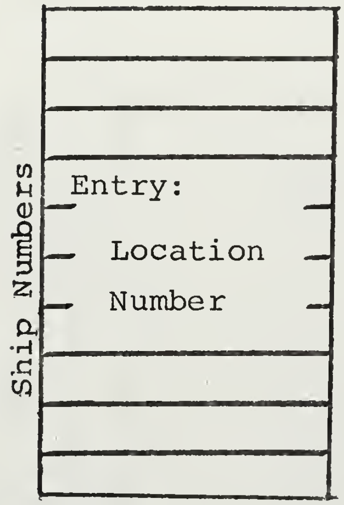
(Permanent and Temporary Copy)



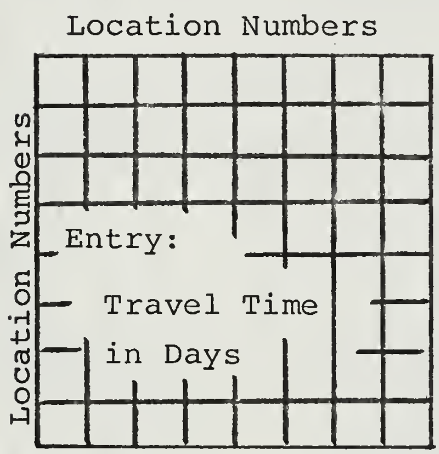
CALENDAR MATRIX



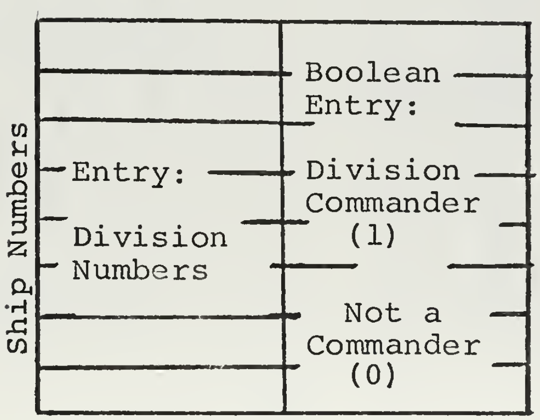
SHIP LOCATION VECTOR



DISTANCE MATRIX



DIVISION MATRIX



SHIP DAY-DUE MATRIX

Ship Number	Entry: Upkeep Date	Entry: Liberty Date	Entry: Outchop Date	Entry: Inchop Date	Entry: Minimum Release Date	Entry: Maximum Release Date

JOB REPORT MATRIX

Job Numbers	Entry: Location Number	Boolean Entry: Active (1) Inactive (0)	Boolean Entry: NOR (1) NAR (0)	Entry: Ship Number on Job	Entry: Group Number	Entry: Minimum Time on Job	Entry: Maximum Time on Job

ACQUIRE FACTOR ELEMENT VALUE MATRICES

UPKEEP ELEMENT

Entry: Number of Days Within Which Upkeep is Planned Or Overdue	Entry: Element Value

LIBERTY ELEMENT

Entry: Number of Days Within Which Liberty is Planned Or Overdue	Entry: Element Value

OUTCHOP ELEMENT

Entry: Number of Days Within Which Outchop is Planned Or Overdue	Entry: Element Value

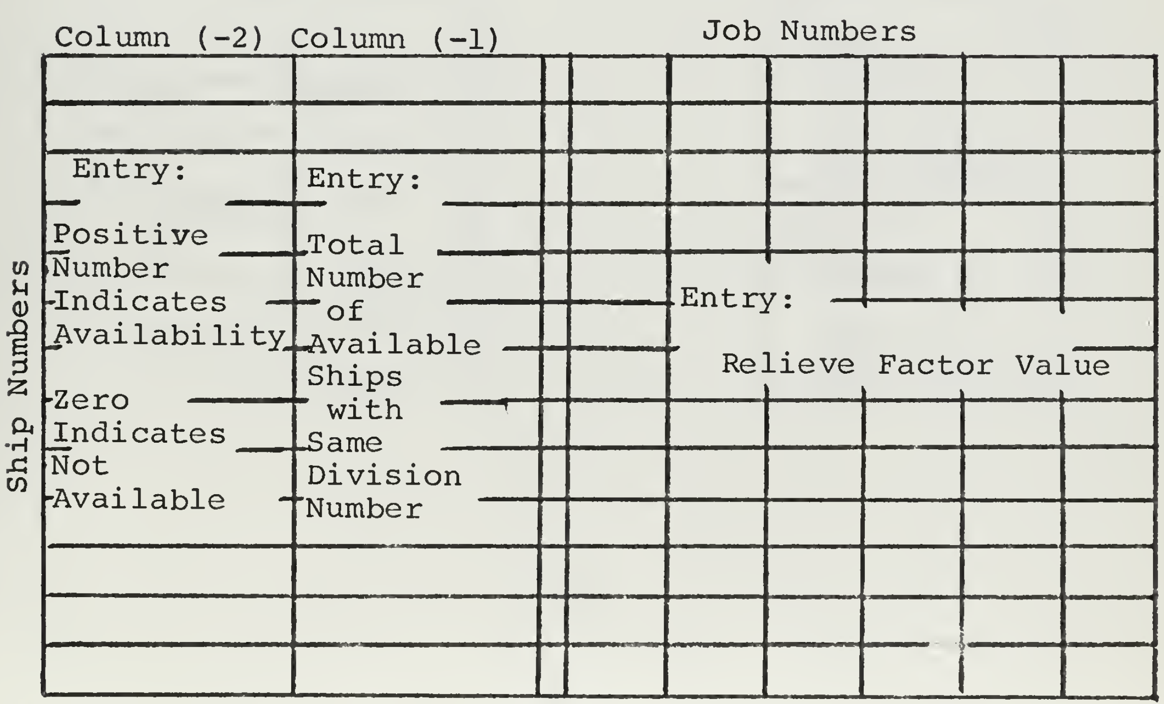
FIRST and FIFTH ELEMENT

Entry: Weighting Factor for Number of Days Travel
Entry: Weighting Factor for Establishing Division Integrity

RELEASE FACTOR ELEMENT VALUE VECTOR

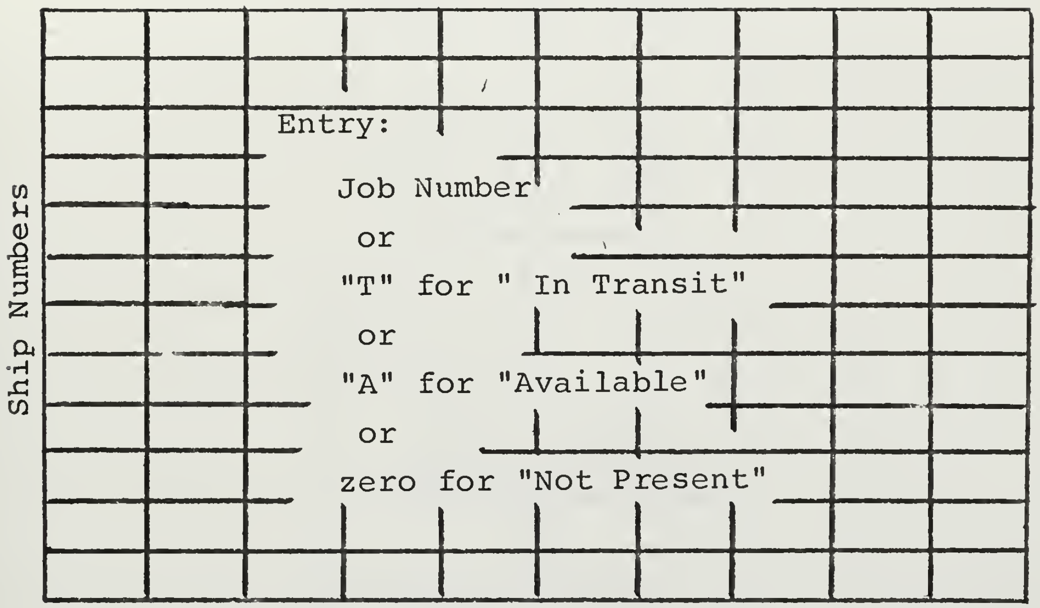
Entry: Weighting Factor for Days Overdue for Relief
Entry: Weighting Factor for Days Overdue for Upkeep
Entry: Weighting Factor for Days Overdue for Liberty
Entry: Weighting Factor for Days Overdue for Outchop

RELIEVE FACTOR MATRIX

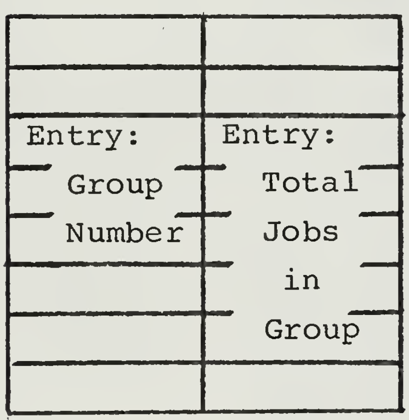


HISTORY MATRIX

Day Numbers 1 - 90

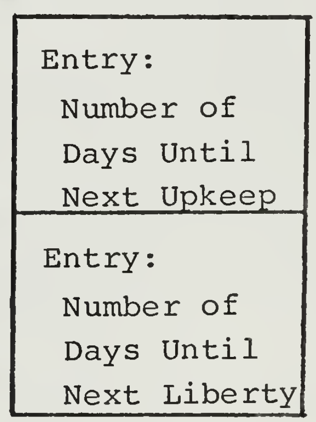


JOB GROUP DATA



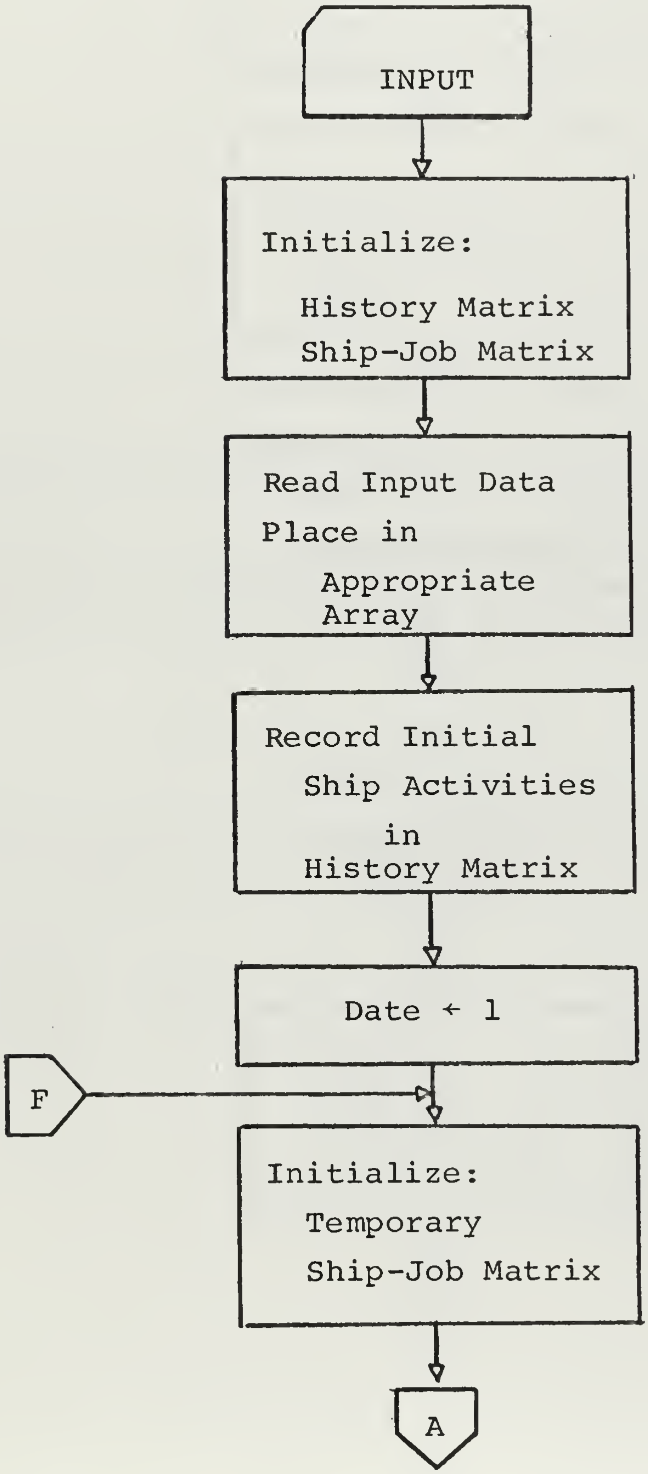
UPKEEP and LIBERTY

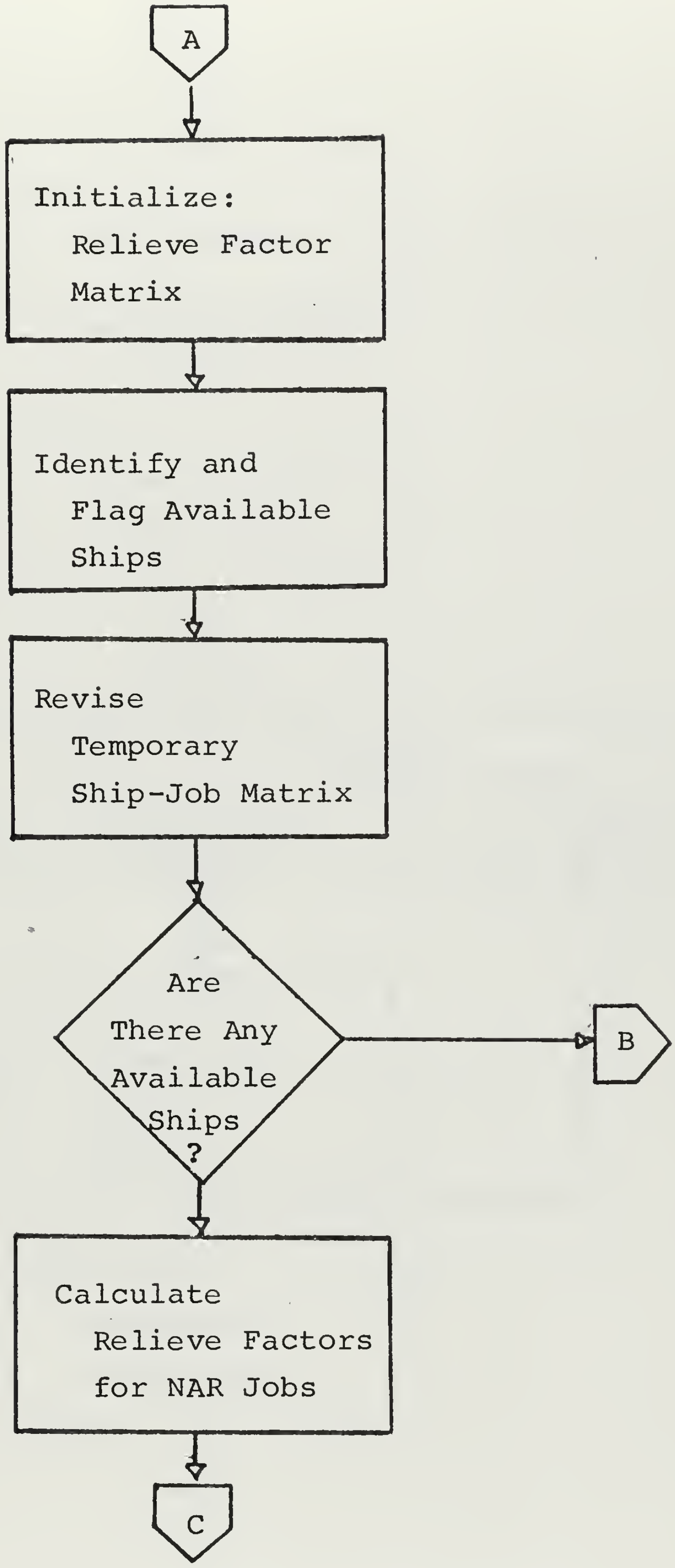
INTERVAL VECTOR

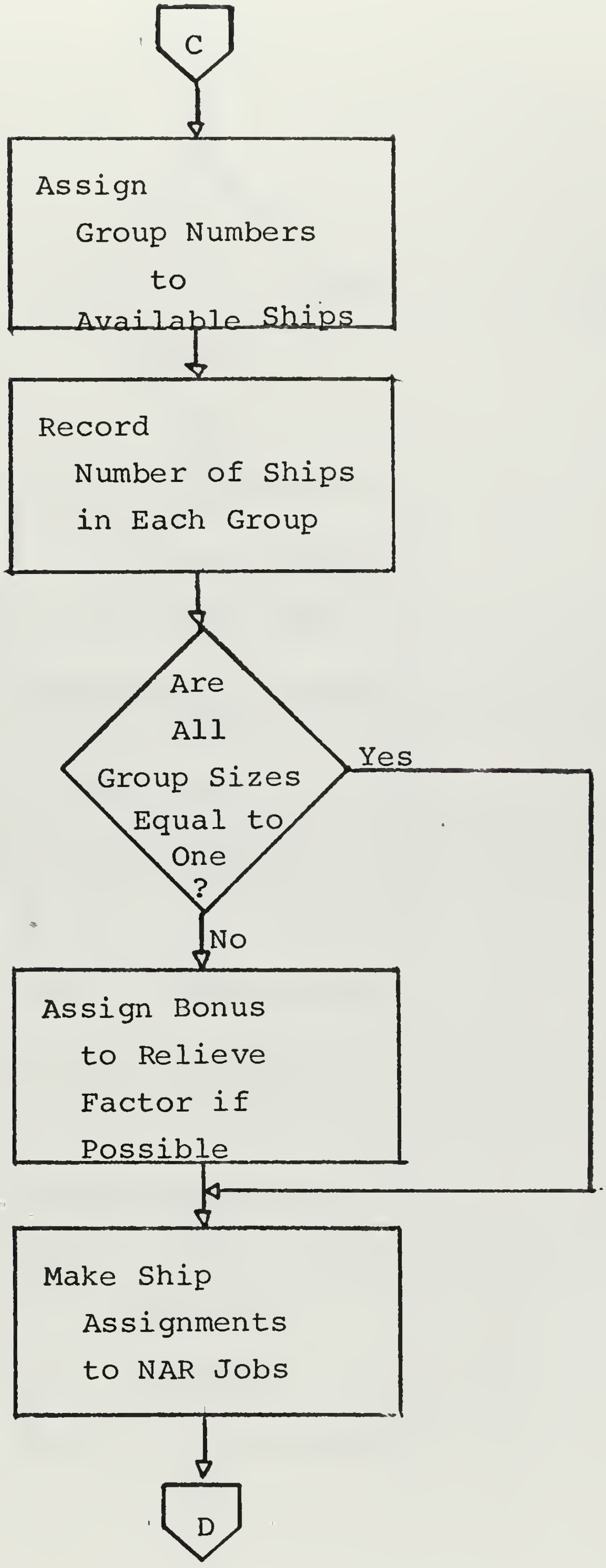


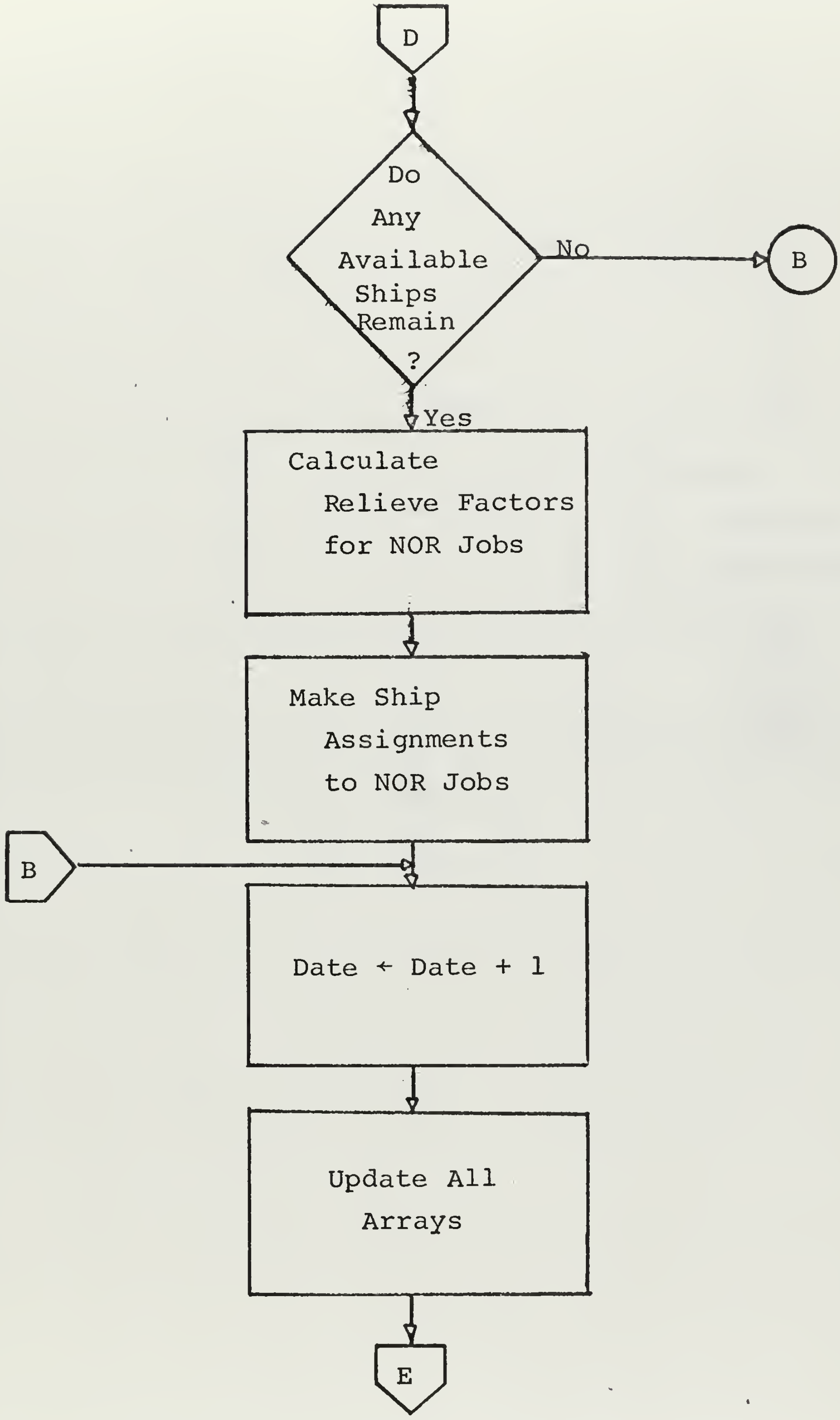
APPENDIX B

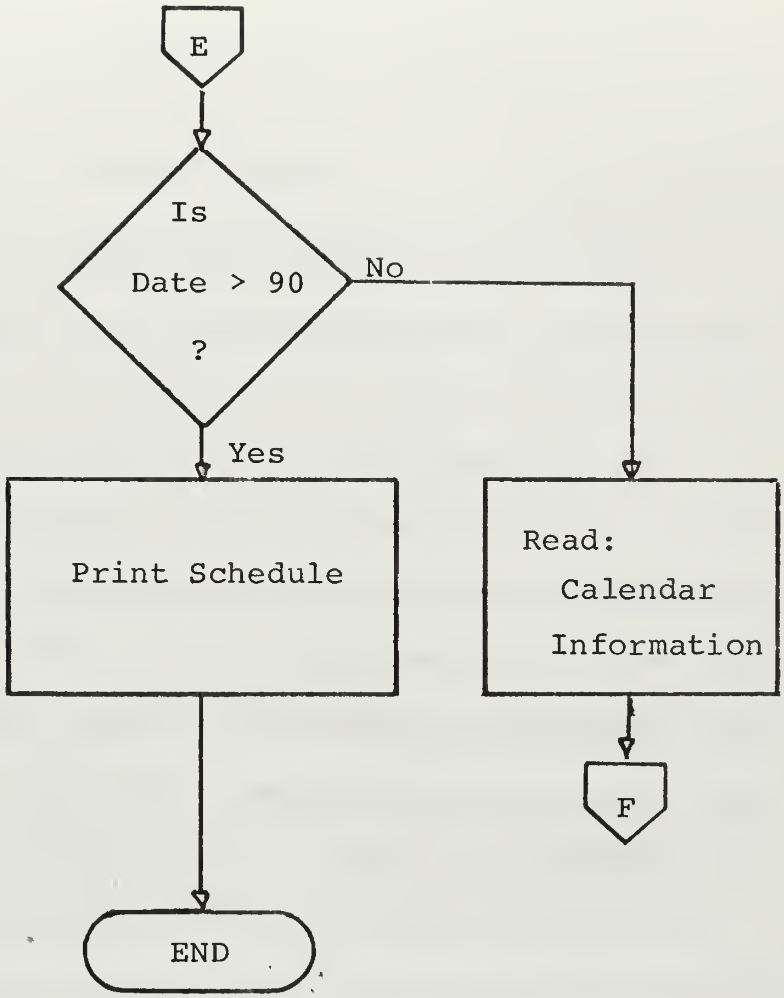
LOGIC FLOW CHART




A
Initialize:
Relieve Factor
Matrix
Identify and
Flag Available
Ships
Revise
Temporary
Ship-Job Matrix
Are
There Any
Available
Ships
?
B
Calculate
Relieve Factors
for NAR Jobs
C


C
Assign Group Numbers to Available Ships
Record Number of Ships in Each Group
Are All Group Sizes Equal to One ?
Yes
No
Assign Bonus to Relieve Factor if Possible
Make Ship Assignments to NAR Jobs
D


D
Do Any Available Ships Remain ?
No
B
Yes
Calculate Relieve Factors for NOR Jobs
Make Ship Assignments to NOR Jobs
B
Date ← Date + 1
Update All Arrays
E


E
Is
Date > 90
?
No
Yes
Print Schedule
Read:
Calendar
Information
F
END

APPENDIX C

IMPLEMENTATION

At the time of the submission of this thesis, the algorithm, as proposed here, has been partially implemented. The system used is the IBM System/360, Model 67, installed at the Naval Postgraduate School. PL/I was chosen as the programming language because language characteristics, such as bit and character string data, allow for ease in handling array manipulations. The storage requirement for a ten ship, ten job model was 88K. Most of the details in program logic have been worked out and enough implementation has been done to determine that the algorithm will generate feasible schedules. Additional programming refinements would be necessary for complete validation of the algorithm's procedure.

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13. ABSTRACT

An algorithm is formulated which schedules ships for at-sea and in-port jobs. The ships are naval warships and their schedules are subject to a variety of constraints. The algorithm allows for individual scheduler preferences and lends itself to future computer implementation.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Scheduling						
Ship Scheduling						
Heuristic Scheduling						